Examining the
Garren-Kirk Dipole Cooling Ring with Realistic Fields

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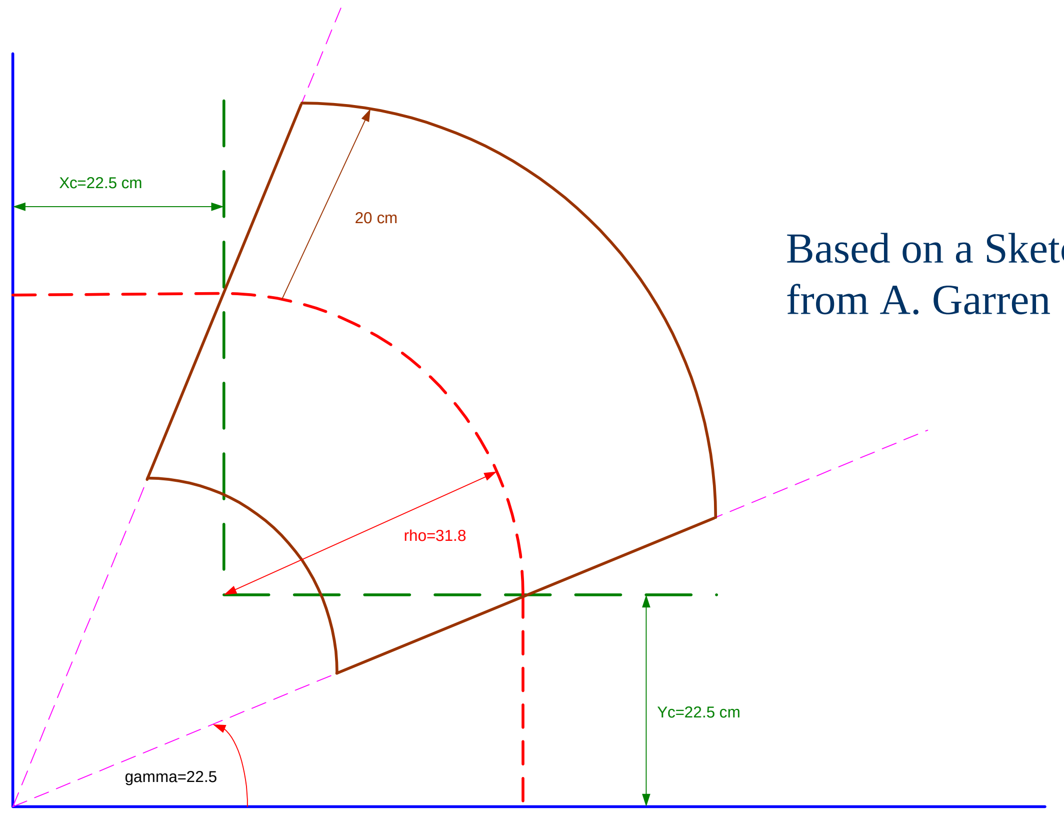
Dipole Ring Parameters

Parameter	Value
Reference Momentum	250 MeV/c
Number of Half-Cells	4
Bend Angle per Half-Cell	90°
Ring Circumference	3.8 m
Number of RF cavities	4
RF Gradient	40 MV/m
Absorber	Pressurized H_2
Hardedge Dipole Field	2.6 T
Straight Length per Half-Cell	40 cm
Dipole Radius of Curvature	31.8 cm

Half Cell Geometry Description



Based on a Sketch from A. Garren

Using TOSCA

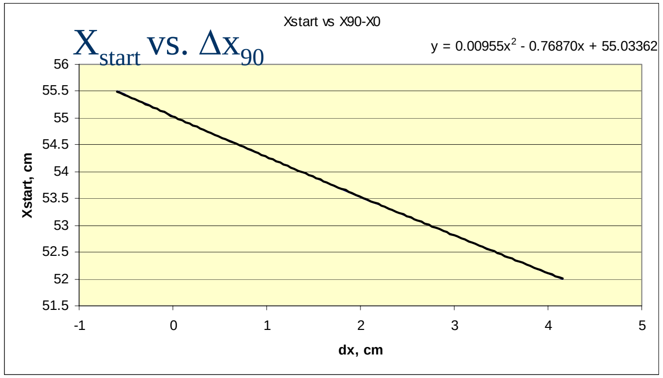
- Hard edge field calculations for the Garren-Kirk Dipole Ring have shown promising results.
  - It is essential to examine the ring using realistic fields that at least obey Maxwell's equations.
- Tosca can supply fields from a coil and iron configuration.
  - We can use the program to supply a field map that can be used by ICOOL and GEANT.
- Tosca itself can also track particles through the magnetic field that it generates.
  - This allows us to avoid the discretization error that comes from field maps.

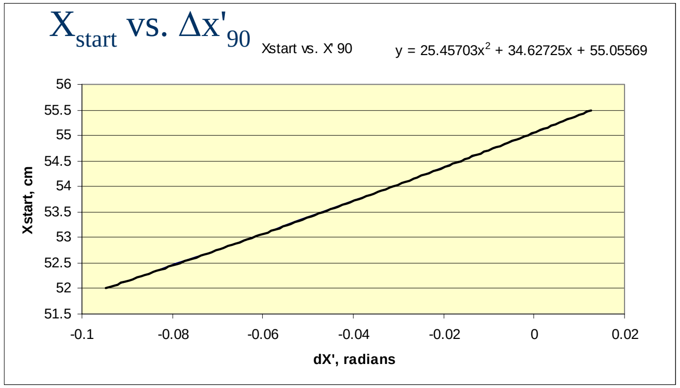
Tosca Model

- For the ease of calculation we are modeling the dipole magnets by its coils only. This may not be the way we would actually engineer the magnet if we actually built it.
  - The field can then be calculated from a Biot-Savart integration directly. No finite-element mesh is necessary if iron is not used.
- There are limitations in the Tosca tracking.
  - Tosca permits only 5000 steps. This limits the step size to ~0.5 mm. This may limit the ultimate precision.

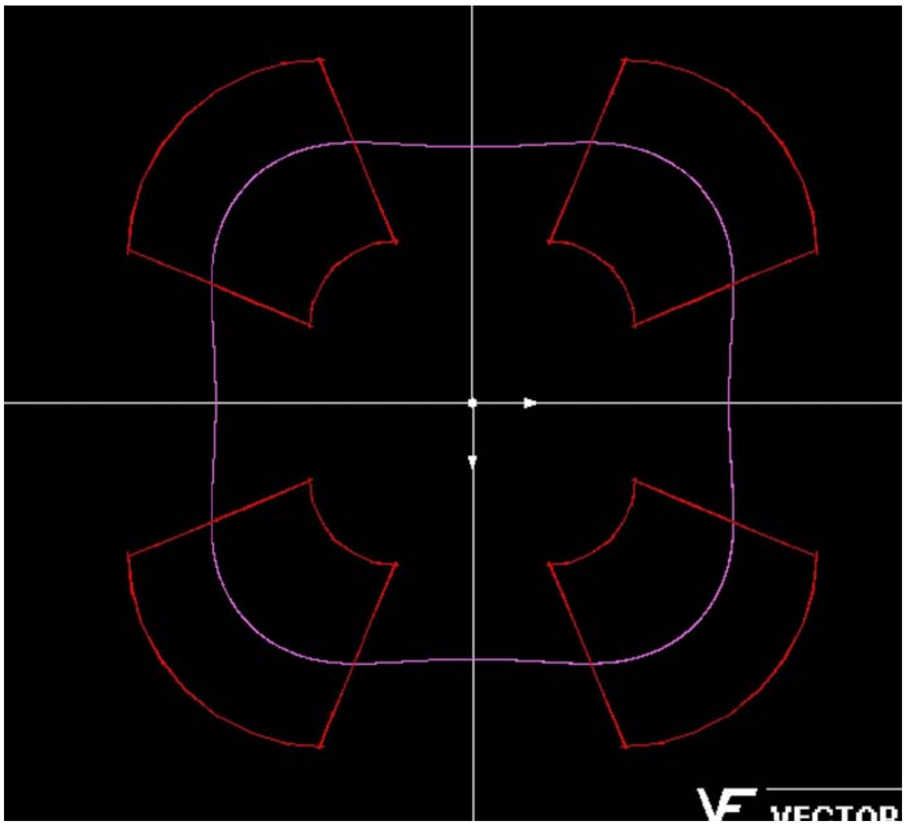
Finding the Closed Orbit

- We know that the *closed orbit* path must be in the *xz plane* and that it must have *x'=0* at the *x-axis* from symmetry.
  - We can launch test particles with different X_{start}.
  - The figures on the right show X_{start} vs. Δx_{90} and X_{start} vs $\Delta x'_{90}$.
    - Where Δx_{90} and $\Delta x'_{90}$ are the variable differences after 90° advance.
    - We find that the best starting values are
      - X_{start}=55.03362 cm for Δx_{90}
      - X_{start}=55.05569 cm for $\Delta x'_{90}$





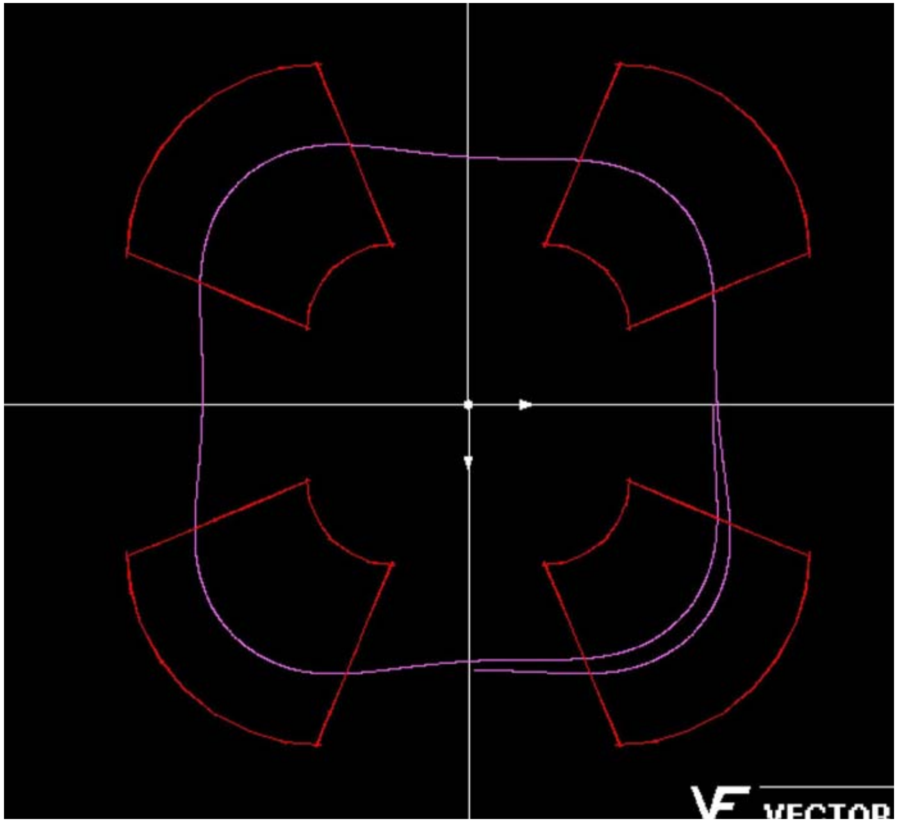
Closed Orbit



Closed orbit trajectory for 250 MeV/c μ started at x=55.02994 cm.

Note that there is curvature in region between magnets since there is still a significant field.

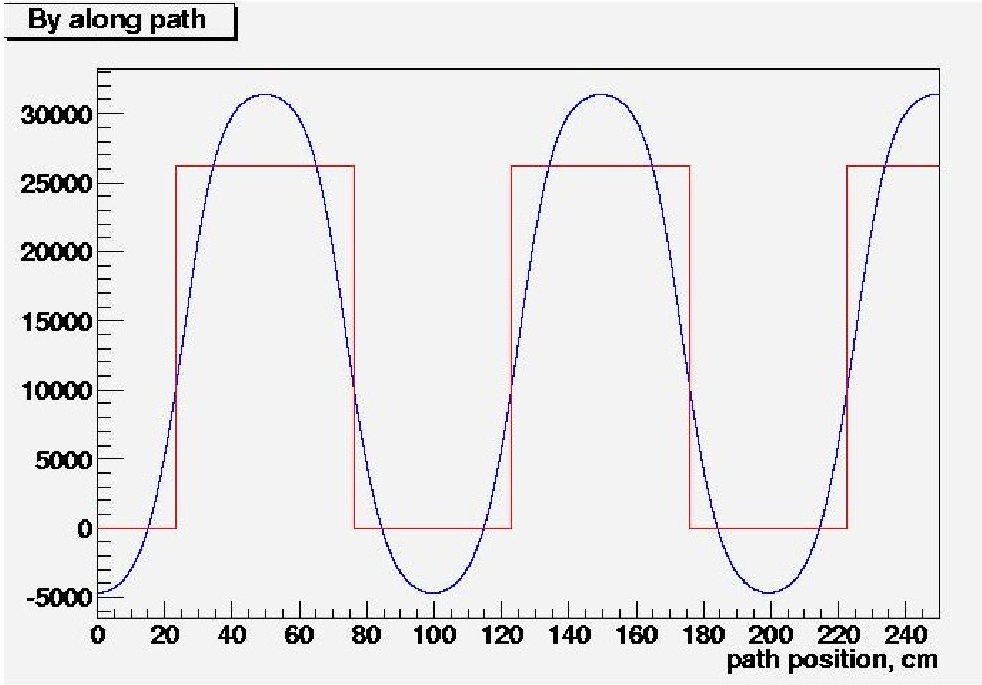
Non-Closed Orbit



Started muon at -2 cm from closed orbit. All other parameters the same.

Field Along the Reference Path

- Figure shows B_y along the 250 MeV/c reference path.
  - The blue curve indicates the field from the Tosca field map.
  - The red curve is the hard edge field.
- Note the –0.5 T field in the gap mid-way between the magnets.



Calculating Transfer Matrices

- By launching particles on trajectories at small variations from the closed orbit in each of the transverse directions and observing the phase variables after a period we can obtain the associated *transfer matrix*.
  - Particles were launched with
    - $\delta x = \pm 1$ mm
    - $\delta x' = \pm 10$ mr
    - $\delta y = \pm 1$ mm
    - $\delta y' = \pm 10$ mr

90° Transfer Matrix

- This is the transfer matrix for transversing a quarter turn:

$$\begin{bmatrix} \delta x \\ \delta x' \\ \delta y \\ \delta y' \end{bmatrix} = \begin{bmatrix} -0.29145 & 31.965 & 0 & 0 \\ -0.0287 & -0.289 & 0 & 0 \\ 0 & 0 & -0.18336 & 52.9949 \\ 0 & 0 & -0.01823 & -0.1853 \end{bmatrix} \begin{bmatrix} \delta x_0 \\ \delta x'_0 \\ \delta y_0 \\ \delta y'_0 \end{bmatrix}$$

- This should be compared to the 2×2 matrix to obtain the twiss variables:

$$\begin{bmatrix} \cos\mu + \alpha \sin\mu & \beta \sin\mu \\ \gamma \sin\mu & \cos\mu - \alpha \sin\mu \end{bmatrix}$$

Twiss Variables Half Way Between Magnets

Parameter	Tosca	A. Garren Synch
μ_x	**98.38°**	**99.8784°**
β_x	**32.3099 cm**	**37.854 cm**
α_x	**-0.00124**	**0**
μ_y	**100.62°**	**92.628°**
β_y	**53.9188 cm**	**56.891 cm**
α_y	**0.0009894**	**0**

Using the Field Map

- We can produce a 3D field map from TOSCA.
  - We could build a GEANT model around this field map however this has not yet been done.
  - We have decided that we can provide a field to be used by ICOOL.
    - ICOOL works in a beam coordinate system.
      - We know the trajectory of the reference path in the global coordinate system.
        - We can calculate the field and its derivatives along this path.

Representation of the Field in a Curving Coordinate System

- Chun-xi Wang has a magnetic field expansion formulism to represent the field in curved (Frenet-Serret) coordinate system.
  - This formulism is available in ICOOL.
  - Up-down symmetry kills off the a_n terms; b_s is zero since there is no solenoid component in the dipole magnets.
  - The b_n(s) are obtained by fitting

    $$B_y(x,s) = \sum b_n(s) x^n$$

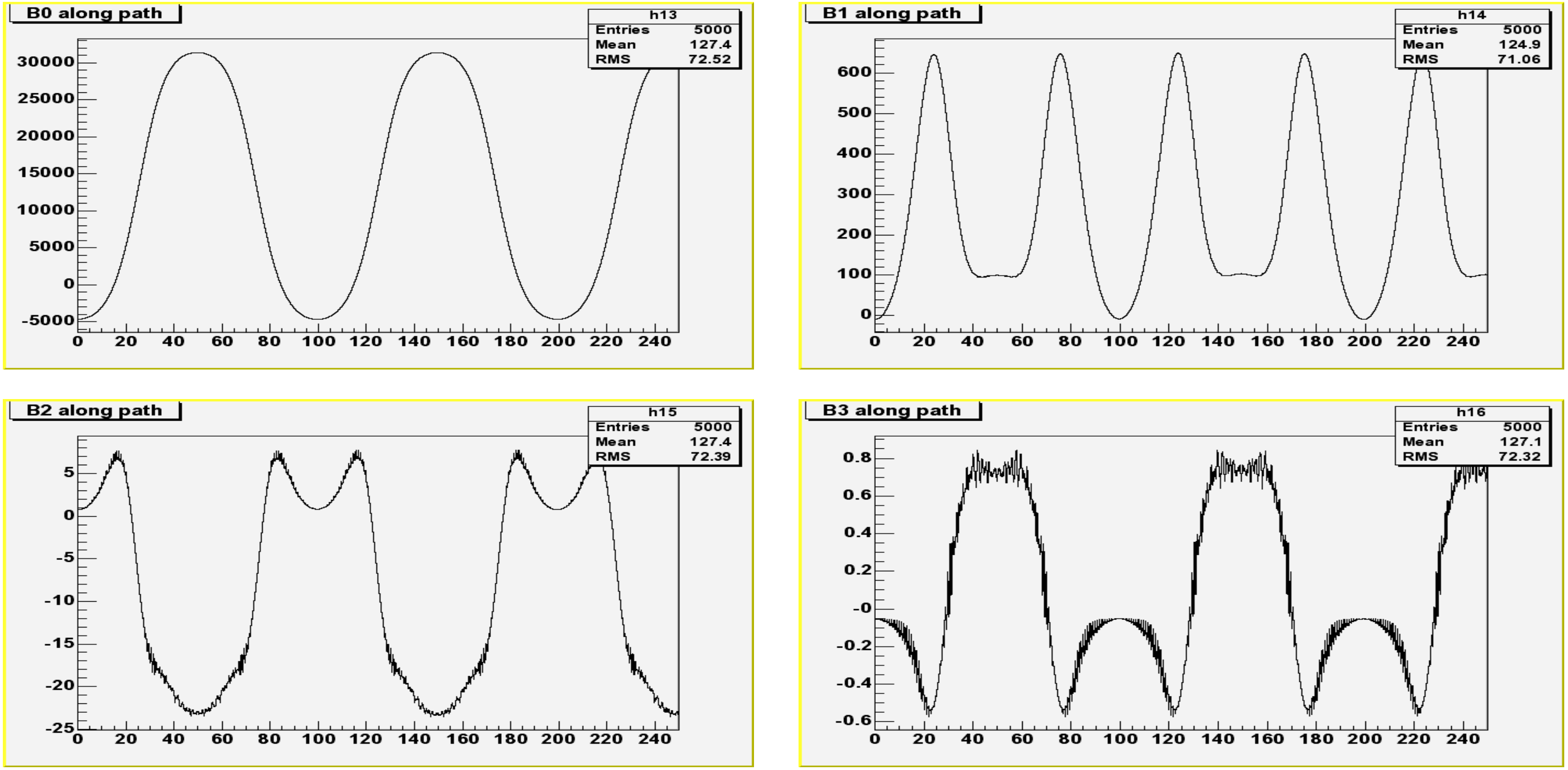
    to the field in the midplane orthogonal to the trajectory at s
    - The field is obtained from a splining the field grid.

$$\begin{aligned} B_x(x,y,s) &= a_1\,x + b_1\,y + a_2\,x^2 + 2b_2\,xy \\ &-\frac{1}{2}\left[2a_2 + \kappa(a_1 - 2b_s') - \kappa' b_s\right] y^2 + a_3\,x^3 + 3b_3\,x^2 y \\ &-\frac{1}{2}\left[6a_3 + a_1'' + 2\kappa(a_2 + 3\kappa' b_s) - 2\kappa^2(a_1 - 3b_s')\right] xy^2 \\ &-\frac{1}{6}\left[6b_3 + b_1'' + 2\kappa(b_2 - b_0'') - \kappa^2 b_1 - \kappa' b_0'\right] y^3 \end{aligned} \tag{26}$$

$$\begin{aligned} B_y(x,y,s) &= b_0 + b_1 x - (a_1 + b_s')\,y \\ &+ b_2\,x^2 - \left[2a_2 + \kappa(a_1 - 2b_s') - \kappa' b_s\right] xy \\ &-\frac{1}{2}\left(2b_2 + b_0'' + \kappa b_1\right) y^2 + b_3\,x^3 \\ &-\frac{1}{2}\left[6a_3 + a_1'' + 2\kappa(a_2 + 3\kappa' b_s) - 2\kappa^2(a_1 - 3b_s')\right] x^2 y \\ &-\frac{1}{2}\left[6b_3 + b_1'' + 2\kappa(b_2 - b_0'') - \kappa^2 b_1 - \kappa' b_0'\right] xy^2 \\ &+\frac{1}{6}\left[6a_3 + 2a_1'' + b_s''' + \kappa(4a_2 + 5\kappa' b_s) - \kappa^2(a_1 - 4b_s')\right] y^3 \end{aligned} \tag{27}$$

$$\begin{aligned} B_s(x,y,s) &= b_s - \kappa b_s\,x + b_0'\,y \\ &+\frac{1}{2}\left(a_1' + 2\kappa^2 b_s\right) x^2 + \left(b_1' - \kappa b_0'\right) xy - \frac{1}{2}\left(a_1' + b_s''\right) y^2 \\ &+\frac{1}{6}\left(2a_2' - 3\kappa a_1' - 6\kappa^3 b_s\right) x^3 + \left(b_2' - \kappa b_1' + \kappa^2 b_0'\right) x^2 y \end{aligned} \tag{28}$$

b_n along the path



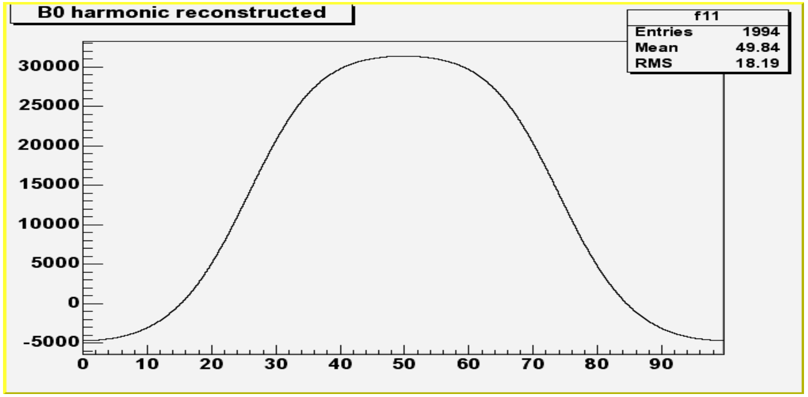
Fourier Expansion of $b_n(s)$

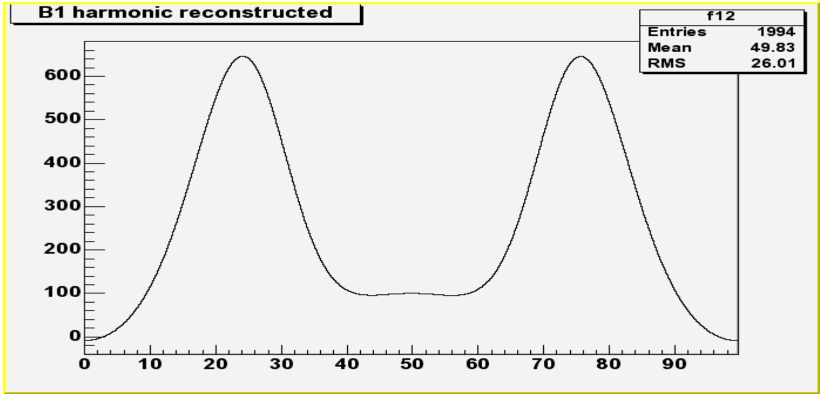
- The $b_n(s)$ can be expanded with a Fourier series:

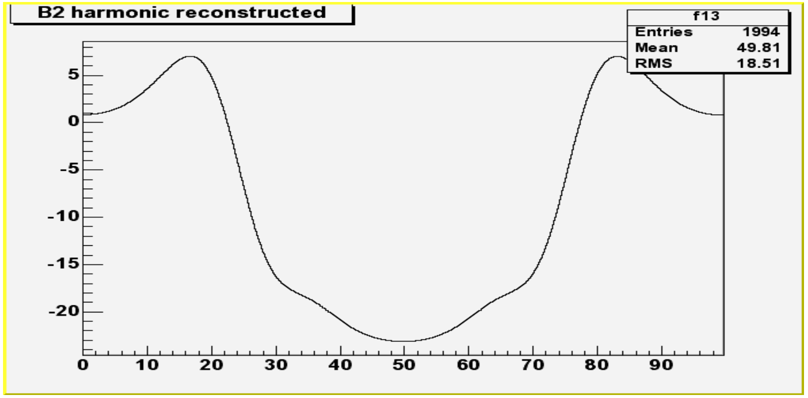
$$b_n = \Re \sum_{k=0}^{N-1} c_{k,n} e^{-ik\frac{s}{T}} \qquad \text{where} \qquad c_{k,n} = \frac{1}{T}\int_0^T b_n(s) e^{ik\frac{s}{T}}$$

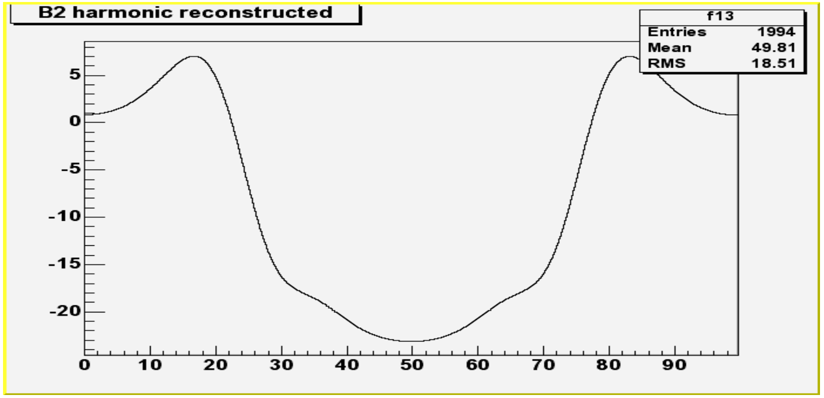
- These Fourier coefficients can be fed to ICOOL to describe the field with the *BSOL 4* option.
- We use the b_n for $n=0$ to 5.

The b_n Series Reconstructed from the $c_{k,\,n}$ Harmonics as a Verification







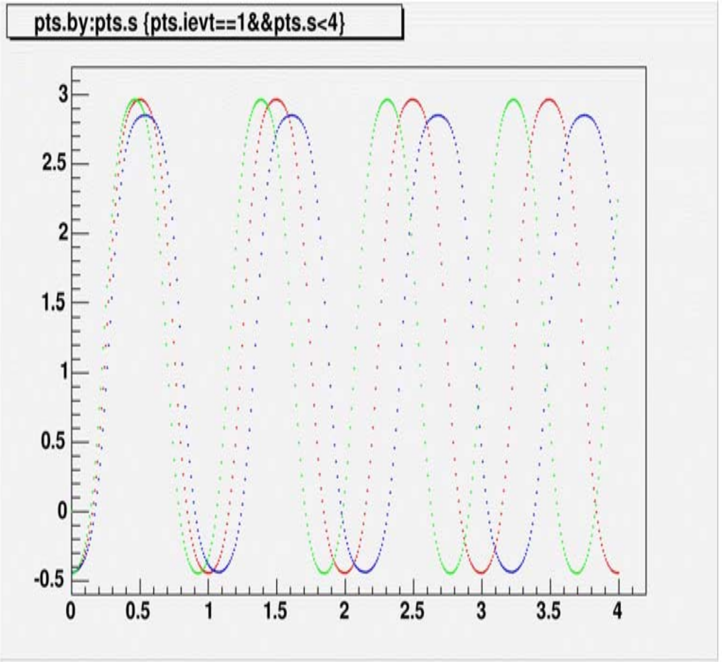


Storage Ring Mode

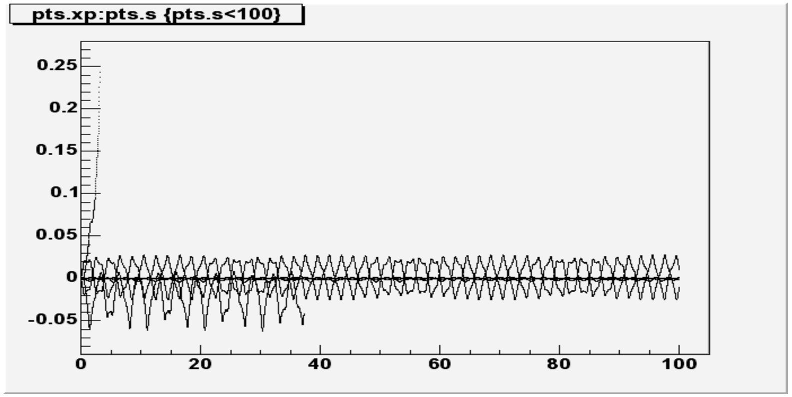
- Modify Harold Kirk's ICOOL deck to accept the Fourier description of the field.
  - Scale the field to 250 MeV/c on the reference orbit.
    - This is a few percent correction.
- Verify the configuration in storage ring mode.
  - RF gradient set to zero.
  - Material density set to zero.
- Use a sample of tracks with:
  - $\delta x=\pm 1$ mm; $\delta y=\pm 1$ mm; $\delta z=\pm 1$ mm;
  - $\delta p_x=\pm 10$ MeV/c; $\delta p_y=\pm 10$ MeV/c; $\delta p_z= \pm 10$ MeV/c;
  - Also the reference track.

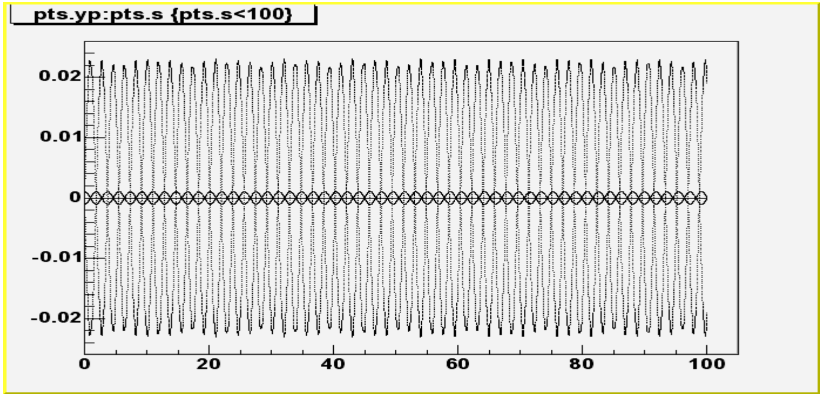
Dipole Field as Seen by ICOOL

- The figure shows the field on a reference orbit as seen by ICOOL.
  - Red is on the reference path
  - Blue is at 5 cm further out
  - Green is at 5 cm closer in
- The curves for ±5 cm reflect the pole face angle. (It is hard to see so trust me.)



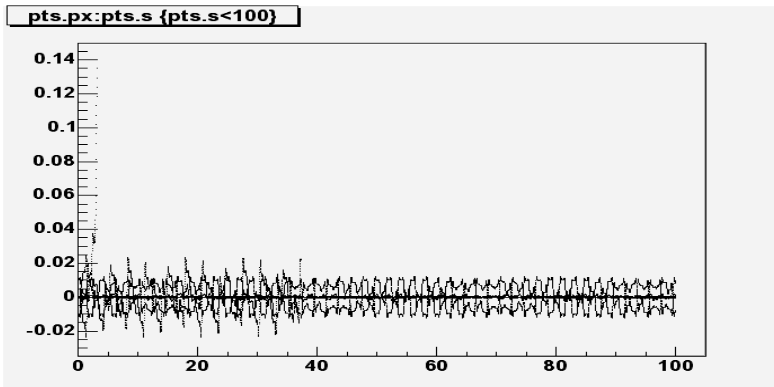
Spacial Deviations in Storage Mode

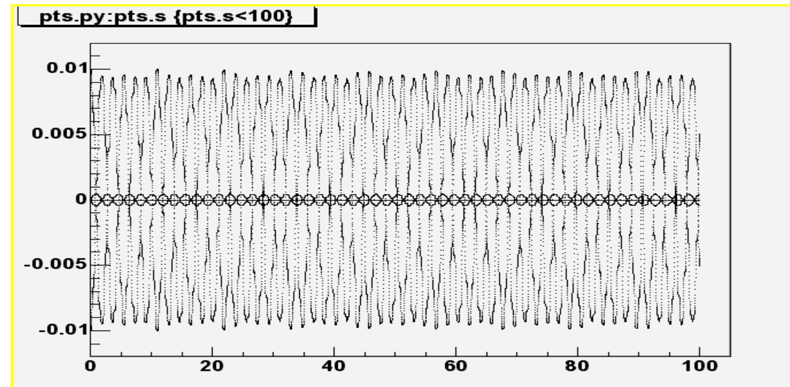




- The figures show δx and δy for the 13 sample tracks.
- Two tracks are lost. The others stay in a range of δx and δy = ±2 cm.
- Most of the track survive >100 m (25 turns).
  - The lost tracks are the two with $\delta p_z = \pm$ 10 MeV/c.

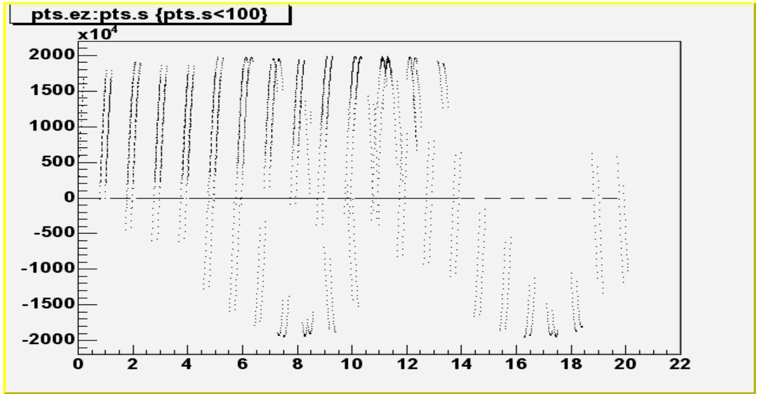
Transverse Momentum Deviation in Storage Ring Mode

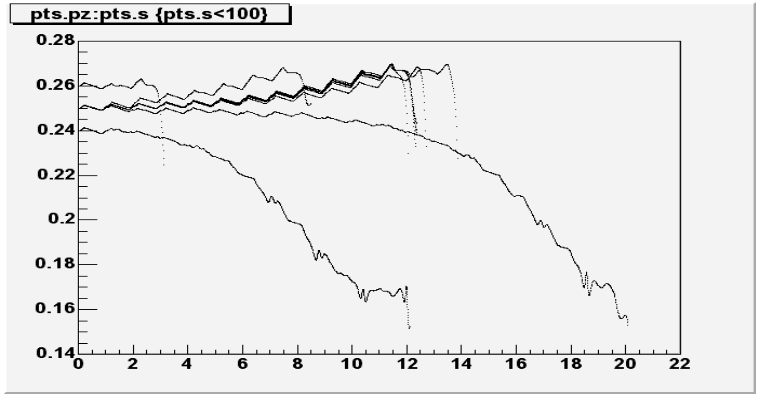




- The figures show δp_x and δp_y deviations.
- They stay in the range ± 10 MeV/c.

Now With RF





- We now set the material of the absorber to gaseous H_2 with 100 atm pressure.
- We need to optimize the RF phase relative to the reference particle.
- We must optimize the RF gradient such that the muon momentum is stable from period to period.
- Figures show plots of E_z and p_z vs. s for gradient of 20 mV/m and phase of 30°

Conclusions

- I am now in the midst of optimizing the case with the RF.
  - I have to understand why I do not do as well as the hard edge model currently does.